Close Target Reconnaissance with Guaranteed Collision Avoidance

Iman Shames[†] [*], Barış Fidan[‡], Brian D. O. Anderson[†]

[†] *Research School of Information Science and Engineering, The Australian National University and NICTA*
[‡] *Mechanical and Mechatronics Engineering Deparment, University of Waterloo, Canada*

SUMMARY

This paper considers the problem of close target reconnaissance by a group of autonomous agents. The overall close target reconnaissance (CTR) involves subtasks of avoiding inter-agent collisions, reaching a close vicinity of a specific target position, and forming an equilateral polygon formation around the target. The agents performing the task fly at constant speed to mimic the velocity behaviour of small fixed-wing unmanned aerial vehicles (UAV). A decentralized control scheme is developed for this overall task and the finite-time convergence of the system under the proposed control law is established. Furthermore, it is guaranteed that no collision occurs among the agent. Relevant analysis and simulation test results are provided.

1. Introduction

Control and coordination of formations of autonomous agents have found many real life civil and defence applications in recent years. The multi-agent autonomous formations can consist of unmanned ground vehicles (UGVs), autonomous underwater vehicles (AUVs), unmanned aerial vehicles (UAVs) or sometimes a combination of more than one agent type. For the special case of formations of UAVs according to [1], the distributed control scheme of a formation of UAVs needs to perform three basic tasks: *(i)* not to let the aircraft hit the ground; *(ii)* not to fly the aircraft beyond their limits; and, *(iii)* not to let any aircraft collide with the others. Furthermore, another task that is as important as the mentioned tasks, in cases where obstacles are present, is not to let any aircraft hit any obstacle during its motion.

When the above motion control tasks are fulfilled the formation of UAVs can accomplish its higher level tasks [2]. For certain tasks, e.g. identification or precise geolocation of a target, the formation may have to spend some time in the proximity of the target location. This task can be accomplished by moving around the target on a desired circle for fixed-wing UAVs flying with the same constant speed while forming an optimal geometry [3] to accomplish the sensing task. It is shown in the literature, e.g. [4, 5], that an optimal sensing geometry in many applications is one corresponding to an equiangular spaced formation around the target, sometimes with each agent at the same distance from the target. An easy way of achieving this geometry is to force agents to form an equilateral polygon around the target. For the case of fixed-wing UAVs, the agents cannot

*Correspondence to: Research School of Information Science and Engineering, The Australian National University, Email: Iman.Shames@anu.edu.au

stop moving; so the task of forming an equilateral polygon around the target changes to forcing the agents to rotate around the target while maintaining an equilateral polygon. For this reason the problem of making the agents form an equilateral polygon while rotating around the target has gained much attention in recent years [6–9].

Along this line of research, [6] has proposed a control framework under cyclic pursuit, causing the agents take up an equilateral polygonal formation moving on a circle whose center is the target. In [7] the problem is addressed via a Lyapunov vector field approach. The solution in [8] relies on invariant set arguments to show that the desired state configuration is the stable equilibrium of the system. However, none of the proposed solutions in the literature to the best knowledge of the authors have considered the problem of collision avoidance between the agents, something that is a real problem in any multiagent formation. Moreover, another problem that is not considered in the proposed solutions, is the *constant speed constraint* for small fixed-wing UAVs, such as Aerosonde UAVs [10]. These constraints had been already considered in [9]. However, the formation structure considered there is limited to a triangular formation with only three agents. In this manuscript, the solution proposed is extended to be applicable to the formations of arbitrary size.

Collision avoidance is another important field of research that has attracted much attention in last few decades. There are two dominant general approaches to achieve collision avoidance in the literature *(i)* planning-based approaches, and *(ii)* behaviour-based approaches. Planning based approaches have their roots in classical control theory, and they are highly dependent on exact mathematical world models. Examples of such approaches towards the problem of obstacle avoidance can be found in [11], [12]. In those works the construction of a special potential function, a navigation function, whose most important property is the absence of undesired local minima is proposed. Implementation of these functions can be found in [13–19]. In these methods the agent follows a calculated path that guarantees that no collision occurs between itself and any obstacles in the environment. The most important assumption made in all the aforementioned works is the availability of an exact model for the environment, which may neither be practical nor realistic in most situations. In most of the behavior-based approaches [20–25] the motion of an agent is determined by taking into account different behaviours, such as avoid-moving-obstacles, avoid-stationary-obstacle, and move-to-goal. These approaches do not usually rely on a strong *a priori* knowledge about the environment, however, they might cause the agent to seek to follow contradicting goals, for example move-to-goal behaviour may force the agent to move in a collision course with an obstacle. Recently, due to much attention being invested in multi-agent systems, collision avoidance methods synthesizing the two approaches have been sought. It is necessary to guarantee the safety of the agents operating in an environment to achieve a goal cooperatively. Because of the challenging nature of multi-agent systems the classical approaches to deal with collision avoidance are rendered impractical, and researchers have moved to solve this problem by taking the virtues of both of the abovementioned approaches and rejecting their vices. For example, the approaches proposed in [26–29] fall in this category; they have elements of both planning-based and reaction-based approaches. In [28, 29] it is analytically proved that the proposed collision avoidance laws achieves its avoidance objective; however, the case where there is a constraint on the motion (upper bounds on speed or acceleration) of the agents is not considered. In this paper we consider a case where such a constraint exists, namely a constant speed constraint, and we prove that the proposed control law guarantees that no collision occurs between the agents.

To summarize, the main contributions of this paper are first, proposing a control strategy that guarantees that no collision happens among the agents while the agents are taking up the desired formation shape in finite time, and second, ensuring the control laws take into account the constant speed constraint.

The outline of the paper is as follows. In the next section the cooperative CTR problem is defined. In Sections 3 and 4 we pose two problems that are directly related to the problem considered in this paper and we propose a solution to each of these problems in the corresponding sections. In Section 5 we propose a solution to the main problem considered in this paper. In the end some concluding remarks and future research directions are presented. Finally in Section 6 two scenarios under nominal performance conditions are simulated to show the performance of the proposed solution. Later in the same section, we consider a case where the motion of the agents is perturbed by wind. Moreover, we show the performance of the method for the case where the speed of the agents are constant but not necessarily equal to each other's.

2. Problem Definition

In this section we formally define the problem of interest in this paper, that is, how to force $N \geq 3$ agents, $A_1, \cdots, A_N$, (which start in essentially arbitrary positions, though remote from the target) to avoid collision with each other and to form an equilateral polygon while rotating around a target of interest. First we present some notational conventions, remarks and assumptions that we use in the rest of this paper.

We denote a circle with center $c \in \mathbb{R}^2$ and radius R by $C(c, R)$ and the corresponding disk by $B(c, R)$, i.e. $B(c, R) = \{x \in \mathbb{R}^2 | \|x - c\| \leq R\}$. The origin $[0, 0]^\top$ of the cartesian coordinate systems for $\mathbb{R}^2$ is denoted by 0_2.

Remark 1. *In Cartesian (x, y, z) coordinates, we assume that our region of interest, R_S, is a ball that lies on the $z = 0$ plane with center at the origin and radius D, $D \in \mathbb{R} > 0$. Each agent is assumed to fly at a constant altitude (equal z-coordinates for all the agents), parallel to R_S. Therefore we consider only the lateral (xy-coordinate) components of the agent kinematics. As a result we design the control laws to steer our agents in 2-dimensional space.*

Assumption 1. *Let $p_i(t) \in \mathbb{R}^2$ denote the position of Agent A_i at time t for each $i \in \{1, \cdots, N\}$. The kinematics of the agent is assumed to be in the single integrator form, i.e.*

$$\dot{p}_i(t) = v_i(t) \tag{1}$$

where $v_i(t)$ is the control signal, satisfying $\|v_i(t)\| = \bar{v}$, $\forall t$ for a pre-specified constant $\bar{v} > 0$.

Assumption 2. *It is assumed that each agent A_i, $i \in \{1, \cdots, N\}$, has a sensing radius of $2D$,where $D \in \mathbb{R} > 0$ is a (large) constant whose value is known a priori, and obtains an identity based on its distance to the origin. The closest agent is termed A_1, the second closest one A_2, and the farthest one A_N. In the case where two or more agents have the same distance to the target, agents with the original smaller labels acquire the smaller labels in the new labeling.*

Assumption 3. *The target of interest is assumed to be located at the origin 0_2.*

Problem 1 (CTR). *Consider N agents, $A_1, \cdots, A_N$ satisfying Assumptions 1 and 2 scattered outside the circle $C(0_2, L_g)$, where $L_g > 0$ is a known constant., $p_i(0) \in B(0_2, D)$. It is desired that the agents (i) avoid collision with each other during motion; (ii) place themselves on the circular orbit $C(0_2, L_g)$; and (iii) rotate in a counterclockwise direction on $C(0_2, L_g)$ while forming an equilateral polygon. The setting is depicted in Fig. 1(a) for $N = 3$.*

To solve this problem we first pose two problems called the Exterior Problem and the Interior Problem, and propose a solution to each of them in the next two sections. Combining the two solutions will allow us to solve Problem 1.

Before elaborating the solution in more details, we first describe the solution that we propose conceptually. The solution is consisted of two main stages. The first stage that we call the Exterior Problem, is concerned with moving the agents inside the circle $C(0_2, L_g)$ without having collision with each other. We guarantee collision avoidance by making sure that the trajectories of the agents are not intersecting and hence eliminate any possibility of collisions. The second stage of the solution deals with the agents that are inside the circle, we call this stage the Interior Problem. At this stage the agents are moved towards some desired destination on the circle $C(0_2, L_g)$ while we guarantee that they will not have collisions by making sure that their trajectories are not intersecting. In the following sections we formalize these arguments and provide analytical proofs for our claims.

3. The Exterior Problem

First we address the problem where the agents, $A_1, \cdots, A_N$, are initially outside $C(0_2, L_g)$ and the goal is to move them inside the circle while guaranteeing that no collision is going to happen among them, and between them and some possible further agents on the circle.

Problem 2 (The Exterior Problem)**.** *Consider N agents, $A_1, \cdots, A_N$ under Assumptions 1 and 2 scattered outside the circle $C(0_2, L_g)$ at time $t = 0$. Assume for all $i \in \{1, \cdots, N\}$, $p_i(0) \in B(0_2, D)$. Consider a further M agents labeled by $\tilde{A}_1, \cdots, \tilde{A}_M$ at positions $\tilde{p}_i(t)$, $i = 1, \cdots, M$, at time t moving in a counterclockwise direction on the circle $C(0_2, L_g)$ with constant speed $\bar{v}$. It is desired that all the agents A_i for $i = 1, \cdots, N$ enter the interior of the circle in finite time without having collision with each other or the $\tilde{A}_i$ for $i = 1, \cdots, M$.*

In order to solve Problem 2, we propose a two stage control law. Define a variable σ_i, which indicates a potential for collision between A_i and any one of the $\tilde{A}_j$, by

$$\sigma_i = \begin{cases} 1 & \exists j \in \{1, \cdots, M\} \text{ such that } F_{i,j^*}(0) < \varepsilon_\alpha \\ 0 & \textit{otherwise} \end{cases}, \tag{2}$$

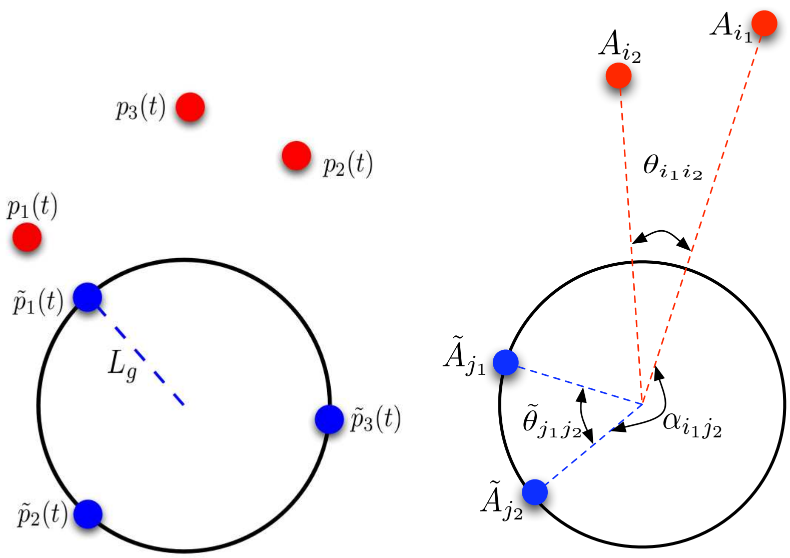
$$F_{i,j^*}(t) = |\Delta L_i(t) - L_g \alpha_{j^* i}(t)| \tag{3}$$

$$\Delta L_i(t) = (|\|p_i(t)\| - L_g|) \mod 2\pi L_g \tag{4}$$

where $\varepsilon_\alpha \ll 2\pi L_g / N$ is a constant positive real number, $\alpha_{ji}(t)$ is defined as the counter clockwise angle from $\tilde{p}_j(t)$ to $p_i(t)$ at time t, as depicted in Fig. 1(b). To explain why σ_i is an indicator of potential for collision, observe that $\|p_i(0)\| - L_g$ is the distance from A_i to the nearest point on the circle, while $L_g \alpha_{ji}(0)$ is the distance around the circle from $\tilde{A}_j$ (at time 0) to the point radially aligned with A_i at the same time. When the difference is very small, there is a risk of collision, since both agents in the same time will reach the vicinity of the point on the circle radially aligned with A_i. We later show that under a mild assumption there is at most one $j^* \in \{1, \cdots, M\}$ such that $F_{i,j^*}(0) < \varepsilon_\alpha$.

If $\sigma_i = 1$, i.e. if there is a potential for collision, agent A_i is controlled at the first stage to move on the circle $C(\bar{p}_i, r_e)$ in clockwise direction, where $r_e = \frac{\Delta_i}{2\pi}$,

$$\Delta_i = -(\Delta L_i(0) - L_g \alpha_{j^* i}(0)) + \varepsilon_\alpha \tag{5}$$



(a) (b)

Figure 1. Geometric Definitions. (a) Positions definitions in the Exterior Problem. (b) The angle settings in the Exterior Problem.

$$\bar{p}_i = -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \nu(p_i(0)) r_e + p_i(0) \tag{6}$$

$$\nu(x) = \begin{cases} \dfrac{x}{\|x\|} & \|x\| \neq 0 \\ 0 & \|x\| = 0 \end{cases}, \quad \forall x \in \mathbb{R}^2 \tag{7}$$

and A_{j*} is the agent which satisfies $|\Delta L_i(0) - L_g \alpha_{j^* i}(0)| < \varepsilon_\alpha$, until it is safe to resume its "normal" trajectory, using the control law

$$v_i(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \nu(p_i(t) - \bar{p}_i)\bar{v}, \quad t \in (0, \bar{\tau}_i), \tag{8}$$

where $\bar{\tau}_i = \dfrac{\Delta_i}{\bar{v}}$. Hence, the first stage of the control law for any agent A_i can be summarized as follows:

$$v_i(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \nu(p_i(t) - \bar{p}_i)\bar{v}, \quad t \in (0, \sigma_i \bar{\tau}_i), \tag{9}$$

Observe that the maximum value of r_e is $\dfrac{\varepsilon_\alpha}{\pi}$, corresponding to $\Delta_i = 2\varepsilon_\alpha$. For $t \geq \sigma_i \bar{\tau}_i$ we use the following second stage control law to control each agent A_i:

$$v_i(t) = -\nu(p_i(t))\bar{v}, \quad t \geq \sigma_i \bar{\tau}_i \tag{10}$$

The only place where $\|v_i(t)\| = 0 \neq \bar{v}$ is at $p_i(t) = 0$, and since the objective of Problem 2 is satisfied (as will be shown later) before this occurs we will not face any problems. Below, we establish an upper bound on the time for which the law (10) applies. Before continuing further we make the following assumption , requiring (reasonably) a minimum angular spacing between the agents $\tilde{A}_i, i = 1, \ldots, M$.

Assumption 4. *Assume that* $\widetilde{\theta}_{ij} L_g > 2\varepsilon_\alpha$, $\forall\, i, j \in \{1, \cdots, M\}$, *where* $\widetilde{\theta}_{ij} \in [0, \pi]$ *is the non-reflex angle between agents* $\tilde{A}_i$ *and* $\tilde{A}_j$ *at time* $t = 0$ *as depicted in Fig. 1(b).*

The reason for introducing this assumption is to eliminate the possibility of having a second potential collision between A_i and any of the $\tilde{A}_j$ (i.e. not just $\tilde{A}_{j^*}$), after doing the evasive maneuver discussed above. In the next proposition we show, first, under Assumption 4 it is not possible for agent A_i to be on a potential collision course to both agents of a pair $(\tilde{A}_j, \tilde{A}_k)$ at any given time t. In other words, each agent can be on a potential collision course with at most one other agent. Second, if at time $t = 0$, A_i is on a collision course with an agent $\tilde{A}_j$, after executing the evasive maneuver introduced earlier, it is not possible for it to be again on a collision course with this *or any other* agent. This means that if an agent evades a potential collision with an agent, this evasion will not cause it to face another potential collision again.

Proposition 1. *Consider the agents* $\tilde{A}_i, \cdots, \tilde{A}_M, A_1, \cdots, A_N$ *in Problem 2 The following statements hold for each agent* A_i, $i \in \{1, \cdots, N\}$*under Assumption 4:*

1. *For all* $t \geq 0$, *there exists no agent pair* $(\tilde{A}_j, \tilde{A}_k)$ *that satisfies both* $F_{i,j}(t) < \varepsilon_\alpha$ *and* $F_{i,k}(t) < \varepsilon_\alpha$
2. *For all* $t \geq \sigma_i \bar{\tau}_i$, *and all* $j \in \{1, ..., M\}$, $F_{i,j}(t) \geq \varepsilon_\alpha$.

Proof. See Appendix I. □

Now can show that each agent A_i enters the region $B(0_2, L_g) \setminus C(0_2, L_g)$ in finite time t_i, as formalized in the next lemma.

Lemma 1. *Consider Problem 2. Each agent* A_i *(*$i \in \{1, \cdots, N\}$*) under the control law (10)enters the region* $B(0_2, L_g) \setminus C(0_2, L_g)$ *in finite time* t_i.

Proof. Consider a positive definite function

$$V_i(p_i(t)) = \frac{1}{2} p_i(t)^\top p_i(t) \tag{11}$$

We have for $t \geq \sigma_i \bar{\tau}_i$

$$\begin{aligned}\dot{V}_i(p_i(t)) &= p_i(t)^\top v_i \\ &= -p_i(t)^\top \nu(p_i(t)) \bar{v} \\ &= -\|p_i(t)\| \bar{v} \\ &\leq -L_g \bar{v}\end{aligned}$$

Therefore A_i enters the interior of the circle $C(0_2, L_g)$ in finite time $t_i \leq \frac{V_i(p_i(0)) - L_g^2/2}{L_g \bar{v}}$ □

Before we continue we make two more assumptions.

Assumption 5. *For all* $i \in \{1, \cdots, N\}$, $\|p_i(0)\| \geq \sqrt{L_g^2 + 4\left(\frac{\varepsilon_\alpha}{\pi}\right)^2}$.

It is reasonable to impose this assumption. It is not desirable for the agents to start too close to the target. Even if they do, a whole new control law approach would be required, but it is not considered here because of the assumed origins of the problem.

Assumption 6. *Let θ_{ij} denote the smallest angle between $p_i(0)$ and $p_j(0)$, as presented in Fig. 1(b). We assume $\forall\, i,j$, $\theta_{ij} \geq 2\sin^{-1}\dfrac{\varepsilon_\alpha}{\varepsilon_\alpha + \pi L_g}$.*

Note here that Assumption 6 can be relaxed by applying a "pre-control" law which forces the agents to rotate around the origin in a counter clockwise motion until the angular separation described in Assumption 6 is satisfied. It is because of the constant speed of the agents and provided that they have different distances from the origin (which can be secured if not initially present through some random perturbations) they have different angular velocity which in turn will result in them achieving the desired angular separation.

The following lemma guarantees that A_i does not have any collision with $\tilde{A}_j$, $j = 1, \cdots, M$, for $0 \leq t \leq t_i$.

Lemma 2. *Consider Problem 2 together with Assumptions 4-6. Any agent A_i, $i = 1, \cdots, N$, controlled by the two stage control law (8)-(10)does not have any collision with $\tilde{A}_j$, $j = 1, \cdots, M$, for $0 \leq t \leq t_i$ under Assumption 5.*

Proof. If $\sigma_i = 1$, because of the control laws and because of Proposition 1 and Assumption 4, A_i moves on a trajectory for a period of $\bar{\tau}_i$ seconds so that at $t = \bar{\tau}_i$, there exists no $\tilde{A}_j$ satisfying $|\Delta L_i(\bar{\tau}_i) - L_g \alpha_{ji}(\bar{\tau}_i)| \leq \varepsilon_\alpha$. Hence A_i will not be on course of collision with any of the agents $\tilde{A}_1, \cdots, \tilde{A}_M$. In order to guarantee that no collision occurs during the time that A_i is undergoing the evasive motion controlled by (8), one appeals to Assumption 5, since A_i needs a circle with maximum radius of ε_α/π of free space, the assumption guarantees that A_i has enough space to complete the evasive motion and will not cross the circle $C(0_2, L_g)$ before finishing the motion. Moreover Assumption 6, guarantees that if an agent is undergoing the evasive maneuver there is enough space for this agent to complete its motion without any other agents potentially goes through its evasive motion trajectory. □

The following lemma guarantees that no collisions occurs between A_i and A_j, $i, j \in \{1, \cdots, N\}$ for $0 \leq t \leq \min(t_i, t_j)$.

Lemma 3. *Consider Problem 2 together with Assumptions 4-6. No collision occurs between the agents A_i, $i = 1, \cdots, N$, before they enter $B(0, L_g) \setminus C(0, L_g)$.*

Proof. Because of the fact that the evasive maneuver either happens at time 0, or does not happen at all, and the biggest circle that can be made because of this evasive motion has a diameter of $2\varepsilon_\alpha/\pi$, it is not possible to have a collision if Assumption 5 holds. □

Proposition 2. *Consider Problem 2 together with Assumptions 4-6. Each agent A_i, $i = 1, \cdots, N$, under control laws (8) -(10) enters $B(0, L_g) \setminus C(0, L_g)$ in finite time t_i without having collision with any other A_j, $i \neq j = 1, \cdots, N$ or any $\tilde{A}_k$, $k = 1, \cdots, M$.*

Proof. The result is a direct consequence of Lemmas 1-3 and Proposition 1. □

4. The Interior Problem

In this section we look at the case where all of the agents are inside $B(0_2, L_g) \setminus C(0_2, L_g)$ and the goal is to force them to move on $C(0_2, L_g)$ forming an equilateral polygon while avoiding any collision with each other. We formally define the problem of interest later in this section.

Before giving the formal problem definition, first consider N fictitious time-varying points, $\tilde{p}_1, \cdots, \tilde{p}_N$ on $C(0_2, L_g)$. Assume these points move on the circle in counterclockwise direction under the kinematic equation

$$\dot{\tilde{p}}_i(t) = \bar{v} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \nu(\tilde{p}_i(t)) \tag{12}$$

Moreover let

$$\tilde{p}_1(0) = \frac{p_1(0)}{\|p_1(0)\|} L_g. \tag{13}$$

and for $i = 2, \cdots, N$ define

$$\tilde{p}_i(0) = \begin{bmatrix} \cos(2(i-1)\pi/N) & \sin(2(i-1)\pi/N) \\ -\sin(2(i-1)\pi/N) & \cos(2(i-1)\pi/N) \end{bmatrix} \tilde{p}_1(0) \tag{14}$$

where $p_1(0) \neq 0$ is the position of an actual agent A_1 inside $C(0_2, L_g)$ at time $t = 0$. This definition and the law (12) ensure that at all times t, the set of points $\tilde{p}_i(t)$ form an equiangular polygon and since each of them has constant speed of $\bar{v}$ they maintain the shape. An example for such setting is presented in Fig. 2.

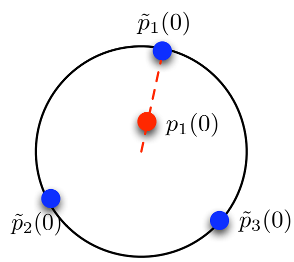


Figure 2. The setting of 3 fictitious points on the circle and their relation with $p_1(0)$ at $t = 0$.

Problem 3 (The Interior Problem)**.** *Consider $m \leq N$ agents, $A_1, \cdots, A_m$ satisfying Assumptions 1 and 2 scattered inside $C(0_2, L_g)$, i.e. for all $i \in \{1, \cdots, m\}$, $p_i(0) \in B(0, L_g) \setminus C(0, L_g)$. It is desired that the agents converge to $\tilde{p}_i$, $i = 1, \cdots, m$ without having any collision with each other.*

Now if we make sure that each agent A_i converges to a moving point on the circle defined by (12) and distinct agents converge to distinct points the problem is solved. To this end first we find the permutation ψ of $\{1, \cdots, m\}$,that minimizes

$$J = \sum_{i \in \{1, \cdots, m\}} \|p_i(0) - \tilde{p}_{\psi(i)}(0)\|. \tag{15}$$

Finding such permutations is studied in optimization literature as an assignment problem, and one can solve such problems using standard algorithms such as the Hungarian algorithm [30].

After calculating the permutation, to simplify the notation, let us relabel $A_1, \cdots, A_m$ such that $p_i(0)$ is assigned to $\tilde{p}_i(0)$), i.e. $\psi(i) = i$.

Now we introduce a rotating coordinate frame with origin at the origin of the global coordinate frame that rotates with the angular speed of $\bar{v}/L_g$ in a counter-clockwise direction. Call this rotating coordinate frame Σ_r. In Σ_r, the points $\tilde{p}_1^r(t) = \tilde{p}_1(0), \cdots, \tilde{p}_N^r(t) = \tilde{p}_N(0)$ are stationary. We use superscript r to show that a variable is in Σ_r, e.g. $p_i^r(t)$ is the position of A_i in Σ_r. In Σ_r the objective is to move A_i to $\tilde{p}_i^r(t) = \tilde{p}_i^r(0)$.. To do so, we have the following control law for controlling A_i, which moves A_i in a straight line in the rotating basis.

$$v_i^r(t) = k_i(t) v_{li}^r(t) \tag{16}$$

where

$$v_{li}^r(t) = \nu(\tilde{p}_i^r(t) - p_i^r(t)) \tag{17}$$

and $k_i(t)$ is chosen so that the absolute speed is $\bar{v}$. In more detail, in the original coordinate frame the control law is

$$v_i(t) = k_i(t) v_{li}(t) + \frac{\bar{v}\|p_i(t)\|}{L_g} v_{ti}(t) \tag{18}$$

where

$$k_i(t) = \bar{v}\sqrt{1 - \frac{\|p_i(t)\|^2}{L_g^2}}, \tag{19}$$

$$v_{ti}(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} v_{ri}(t), \tag{20}$$

$$v_{ri}(t) = -\nu(p_i(t)). \tag{21}$$

The following lemma establishes that A_i converges to its desired position and then starts rotating around the target with a constant speed.

Lemma 4. *Consider Problem 3. Under the control laws (17)-(21) each agent A_i is guaranteed to converge to its desired position $\tilde{p}_i$ in finite time.*

Proof. Note that $v_{li}^r(t) = \nu(\tilde{p}_i^r(t) - p_i^r(t))$ is constant, furthermore, for all t by definition of the origin of Σ_r, $\|p_i(t)\| = \|p_i^r(t)\|$. Therefore we have

$$p_i^r(t) = p_i^r(0) + \bar{p}_i(t) v_{li}^r(t)$$

where $\bar{p}_i(t) \geq 0$ is a scalar and

$$\dot{p}_i^r(t) = \dot{\bar{p}}_i(t) v_{li}^r(t) = k_i(t) v_{li}^r(t)$$

Thus

$$\dot{\bar{p}}_i(t) = k_i(t) = \bar{v}\sqrt{1 - \frac{\|p_i^r(t)\|^2}{L_g^2}} = \bar{v}\sqrt{1 - \frac{\|p_i^r(0) + \bar{p}(t) v_{li}^r(t)\|^2}{L_g^2}}.$$

Define $p_0^r = \|p_i^r(0)\|$ and let the acute angle formed by $p_i^r(0)$ and $v_{li}^r(t)$ be γ, see Fig. 3. Then:

$$\begin{aligned} \|p_i^r(0) + \bar{p}(t) v_{li}^r(t)\|^2 &= (p_0^r)^2 + (\bar{p}_i(t))^2 + 2\bar{p}_i (v_{li}^r)^\top p_i^r(0) \\ &= (p_0^r)^2 + (\bar{p}_i(t))^2 + 2\bar{p}_i p_0^r \cos\gamma \\ &= (p_0^r \sin\gamma)^2 + (p_0^r \cos\gamma + \bar{p}_i(t))^2 \\ &= L_g^2 - A^2 + (p_0^r \cos\gamma + \bar{p}_i(t))^2. \end{aligned}$$

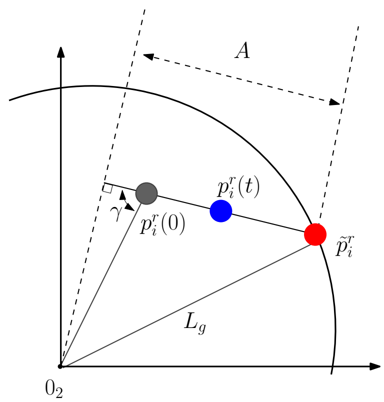


Figure 3. The variables as described in the proof of Lemma 4.

Hence

$$\dot{\bar{p}}_i(t) = \bar{v}\sqrt{1 - \frac{L_g^2 - A^2 + (p_0^r\cos\gamma + \bar{p}_i(t))^2}{L_g^2}}$$
$$= \bar{v}\sqrt{\frac{A^2 - (p_0^r\cos\gamma + \bar{p}_i(t))^2}{L_g^2}}$$
$$= \frac{\bar{v}A}{L_g}\sqrt{1 - \left(\frac{p_0^r}{A}\cos\gamma + \frac{\bar{p}_i(t)}{A}\right)^2}$$

Thus

$$\bar{p}_i(t) = A\sin\frac{\bar{v}(t-t_0)}{L_g} - p_0^r\cos\gamma,$$

where $t_0 = L_g\sin^{-1}(p_0^r\cos\gamma/A)/\bar{v}$. This implies that at $t = \frac{\pi L_g}{2\bar{v}} + t_0$, $\bar{p}_i(t) = A - p_0^r\cos\gamma = \|\tilde{p}_i^r(0) - p_i^r(0)\|$, i.e. $p_i^r(t) = \tilde{p}_i^r$. □

The following lemma guarantees that A_i does not have any collision with any other agent A_j.

Lemma 5. *Agent A_i controlled by the control law (17)-(21) does not have any collision with any other agent A_j.*

Proof. We show that no collision is possible through geometric reasoning. Let us look at the agents in Σ_r. We assumed $\tilde{p}_i$ is chosen in a way such that $\sum_i \|p_i(0) - \tilde{p}_i(0)\|$ is minimum. The control law in Σ_r ensures that each of the agents A_i, $i = 1, \cdots, N$ moves on a straight line that connects $p_i^r(t)$ and $\tilde{p}_i(0)$. In order to have a possibility of collision these lines should intersect. To obtain a contradiction assume that $\|p_i^r(0) - \tilde{p}_i^r(0)\| + \|p_j^r(0) - \tilde{p}_j^r(0)\|$ is minimum and the lines connecting $p_i^r(0)$ and $p_j^r(0)$ to $\tilde{p}_i^r(0)$ and $\tilde{p}_j^r(0)$ respectively have an intersection at point P. Writing the triangle inequality in the triangles $\triangle p_i^r(0)P\tilde{p}_j^r(0)$ and $\triangle p_j^r(0)P\tilde{p}_i^r(0)$ we have

$$\|p_i^r(0) - P\| + \|\tilde{p}_j^r(0) - P\| \geq \|p_i^r(0) - \tilde{p}_j^r(0)\|$$

$$\|p_j^r(0) - P\| + \|\tilde{p}_i^r(0) - P\| \geq \|p_j^r(0) - \tilde{p}_i^r(0)\|.$$

Summing these equalities we have

$$\|p_i^r(0) - P\| + \|\tilde{p}_j^r(0) - P\| + \|p_j^r(0) - P\| + \|\tilde{p}_i^r(0) - P\| \geq \|p_i^r(0) - \tilde{p}_j^r(0)\| + \|p_j^r(0) - \tilde{p}_i^r(0)\|$$
$$\|p_i^r(0) - \tilde{p}_i^r(0)\| + \|p_j^r(0) - \tilde{p}_j^r(0)\| \geq \|p_i^r(0) - \tilde{p}_j^r(0)\| + \|p_j^r(0) - \tilde{p}_i^r(0)\|$$

which means there is another sum of distances that is smaller than the current one and hence we obtain a contradiction to the requirement that $\sum_i \|p_i^r(t) - \tilde{p}_i(0)\|$ be minimum. Hence in Σ_r the trajectories of no pair A_i and A_j have intersection. Going back to the original coordinate frame, since each pair of the points at time t in the trajectories in Σ_r are rotated by the same angle again no intersection happens. Hence, no collision is possible under the the control law (17)-(21) . □

Proposition 3. *Problem 3 is solved in finite time if each agent A_i is controlled by control laws (18)-(21).*

Proof. The proof is a consequence of Lemmas 4 and 5. □

5. Proposed Solution to CTR Problem

We propose a two level control scheme to solve Problem 1. The first level guarantees that the agents $A_2, \dots, A_N$ enter $B(0, L_g) \setminus C(0, L'_g)$, where $L'_g = L_g - \delta_g$ for some positive $\delta_g << L_g$, without having collision with each other, in finite time, and the second level guarantees that the agents start rotating around the target, while constructing an equilateral polygon, asymptotically. We propose Algorithm 1 to solve this problem. As mentioned earlier the solution consists of solving two subproblems. Hence, it is required to switch from the control laws to solve the Exterior Problem to the laws that solve the Interior Problem for each agent A_i at the time when A_i enters $B(0_2, L'_g)$, i.e. t_i. In Algorithm 1 this occurs when the *mode* changes from *exterior* to *interior* and the condition for such a change is $p_i(t)$ becoming less than L'_g. Due to the laws for the Interior Problem, there is no potential chattering.

For agent A_1, which is the only exception for application of the two level control scheme, we propose the control law (10) with $\sigma_1 = 0$ till A_1 enters the region $B(0_2, L'_g)$, and using (18)-(21) after that.

In the following lemma we establish that agent A_1 enters the region $B(0_2, L'_g)$ in finite time.

Lemma 6. *Agent A_1 controlled by the control law (10) enters the region $B(0_2, L'_g)$ in finite time t_1.*

Proof. The proof is identical to that of Lemma 1 and is omitted. □

For agents $A_2, \dots, A_N$ it is assumed that they are controlled by control laws (9)-(10)when they are outside $B(0, L'_g)$, and with (18)-(21) when they are inside, avoiding collision with any $\tilde{p}_i(t)$ defined by (14). So the problem comes down to solving Problems 2 and 3 consecutively. We propose Algorithm 1 to address Problem 1. However, first we introduce the following relationship of δ_g and ε_a:

$$\varepsilon_a = L_g \sin^{-1}\left(\frac{\sqrt{b^2 + \delta_g^2}}{L_g}\right) \tag{22}$$

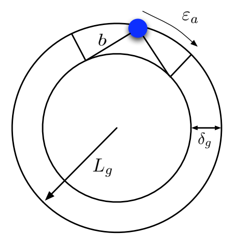


Figure 4. The relationship between b, δ_g and ε_a.

where b is the length of the tangent line segment from any of $\tilde{p}_i$ to the circle $C(0_2, L'_g)$. This setting is depicted in Fig. 4. This relationship guarantees that the agents starts with a nonzero linear speed when they are inside the circle and can move towards their desired destination without collision.

Algorithm 1 Close Target Reconnaissance with Collision Avoidance Guarantee for Agent A_ℓ $(\ell \in \{1, \cdots, N\})$

Initialize time: $t \leftarrow 0$
Label N Agents 1 to N such that; $\|p_1(0)\| \leq \cdots \leq \|p_N(0)\|$
$\tilde{p}_1(0) \leftarrow \frac{p_1(0)}{\|p_1(0)\|} L_g$
$mode \leftarrow exterior$
for $i = 1$ to N-1 **do**
  $\tilde{p}_i(0) \leftarrow \begin{bmatrix} \cos(2(i-1)\pi/N) & \sin(2(i-1)\pi/N) \\ -\sin(2(i-1)\pi/N) & \cos(2(i-1)\pi/N) \end{bmatrix} \tilde{p}_1(0)$
end for
if $\sigma_\ell = 1$ **then**
  while $t \leq \bar{\tau}_\ell$ **do**
    $v_\ell(t) \leftarrow \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \nu(p_\ell(t) - \bar{p}_\ell)$, as defined in (8)
  end while
end if
while $\|p_\ell(t) - \tilde{p}_{\psi(\ell)}(t)\| > \delta$ `/*` δ `is a small positive real number */` **do**
  if $mode = exterior$ **and** $\|p_\ell(t)\| \leq L'_g$ **then**
    $mode \leftarrow interior$
  end if
  if mode=exterior **then**
    $v_\ell(t) \leftarrow -\bar{v}\nu(p_\ell(t))$ as defined in (10)
  else
    if Any new agent has entered $B(0_2, L'_g)$ **then**
      $\phi \leftarrow$ Set of agents in $B(0_2, L'_g)$
      $m \leftarrow |\phi|$
      Assume $\phi = \{1_\phi, \cdots, m_\phi\}$. Find the bijection $\psi^m(i_\phi)$, $i_\phi \in \phi$, from ϕ to $\{1, \cdots, m\}$, such that $\sum \|\tilde{p}_{\psi(i_\phi)} - p_{i_\phi}\|$ is minimum.
    end if
    $v_\ell(t) \leftarrow k_\ell(t) v_{l\ell}(t) + \frac{\bar{v}\|p_{i\ell}(t)\|}{L_g} v_{t\ell}(t)$ as defined in (18)
  end if
end while

Theorem 1. *Under Assumptions 1-6, Algorithm 1 solves Problem 1 in finite time.*

Proof. The proof is a result of Proposition 2, Proposition 3, and Lemma 6.

Proposition 2 and Lemma 6 guarantee that each A_i, $i = 1, \cdots, N$ enters $C(0_2, L'_g)$ without having collision with other agents outside or possibly moving on the circle. Proposition 3 guarantees that each agent inside the circle converges to a desired time-varying point on the circle defined by (12)-(14) without having collision with other agents.

In the end the constant speed condition is satisfied while the agents are being controlled by any of the control laws. □

Remark 2 (Robustness of the Proposed Control Scheme)**.** *Robustness of the systems with exponential convergence to external disturbances and noise is a known fact, e.g. see [31]. In the light of the fact that the control law that is proposed in this section converges to the desired solution in finite-time, which in fact is faster than exponential convergence, the known robustness properties of exponential systems are carried over to the proposed control strategy.*

Remark 3. *If the constant speed constraint is relaxed, the control strategy introduced here can be easily extended to solve the CTR problem for a moving target with known velocity in finite time. We will present a simulation result considering this scenario in the next section.*

5.1. Information Exchange among the Agents

Algorithm 1 does not require any knowledge about the trajectory of all the other agents all the time. However, it requires the knowledge of other agents positions at a finite number of time instants. Initially, each agent can measure the positions of all the other agents at time $t = 0$. Later, at times $t = t_i$, $i = 2, \cdots, N$, all the agents inside $B(0_2, L_g)$ know the position of all the other agents that are inside $B(0_2, L_g)$ as well. At other times knowing the position of other agents is not necessary.

5.2. Time Required to Accomplish CTR

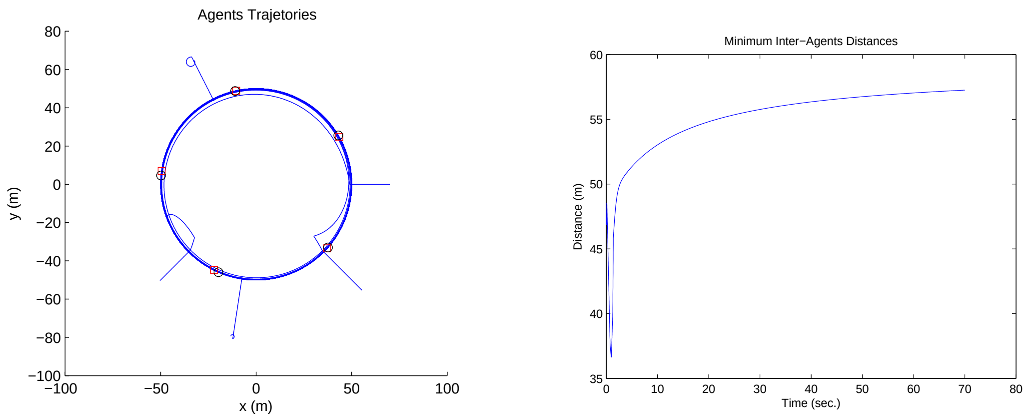
Any agent A_i requires t_i units of time to enter $B(0_2, L'_g)$, see Lemma 1, and $\bar{t}_i = \dfrac{\pi L_g}{2\bar{v}} + \bar{t}_0$ where $\bar{t}_0$ has a maximum equal to $L_g\pi/2\bar{v}$, see Lemma 4. Hence the total time for an agent to go to its desired position is $t_i + \dfrac{\pi L_g}{\bar{v}}$, thus the minimum time required to accomplish CTR is $\max\limits_{i=1,\cdots,N} t_i + \dfrac{\pi L_g}{\bar{v}}$.

One may be interested in decreasing such a time, to do so the obvious way is to increase the value of $\bar{v}$, or deploy the agents very close to the $B(0_2, L'_g)$ in order to reduce the magnitude of t_i.

6. Simulation Results

In this section we introduce some simulation results to show the performance of the Algorithm 1. The values that are used in these simulations can be scaled to more realistic numbers, however the issue here is to show the applicability of the algorithms.

Scenario 1: Satisfying all the assumptions and no disturbance. In the first scenario we consider the case where $N = 5$, $L_g = 50m$, and $\bar{v} = 30m/s$. Agents trajectories are depicted in Fig. 5(a), and the minimum of all inter-agent distances at each time is depicted in Fig. 5(b). As



(a) The agent trajectories. The red squares represent the final position of the agents after 70 seconds of simulation time

(b) Minimum inter-agent distances.

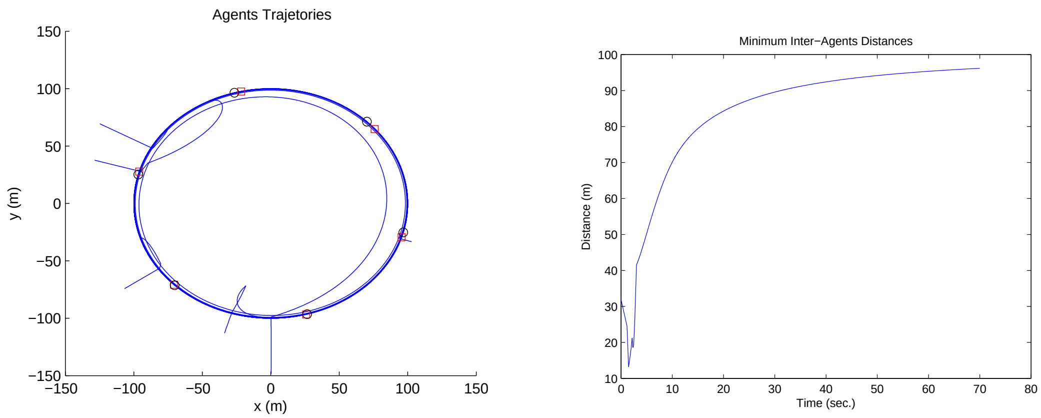
Figure 5. Scenario 1: $N = 5$, $L_g = 50m$, $\bar{v} = 30m/s$

is evident from the figures the agents form an equilateral polygon and avoid having collision with each other.

Scenario 2: Not satisfying all the Assumptions 5 and 6 and no disturbance. In the second scenario we consider the case where $N = 6$, $L_g = 100m$, and $\bar{v} = 30m/s$, but agent positions do not satisfy Assumptions 5 and 6. Agent trajectories are shown in Fig. 6(a), and the minimum of all inter-agent distances at each time is depicted in Fig. 6(b). As is evident from the figures the agents form an equilateral polygon and avoid collisions with each other.

Scenario 3: Not satisfying all the Assumptions 5 and 6 and with disturbance. In the third scenario we consider the same case as considered in the previous scenario, with the additional consideration of the influence that wind might have on the motion of the agents. We have modeled wind as a Gaussian variable with zero mean and 1 $(m/s)^2$ variance that affects the velocity of the agents individually. Agent trajectories are shown in Fig. 7(a), and the minimum of all inter-agent distances at each time is depicted in Fig. 7(b). As is evident from the figures the agents form an equilateral polygon and avoid collisions with each other. Scenario 3 is a demonstration of the robustness of the algorithm to the assumptions.

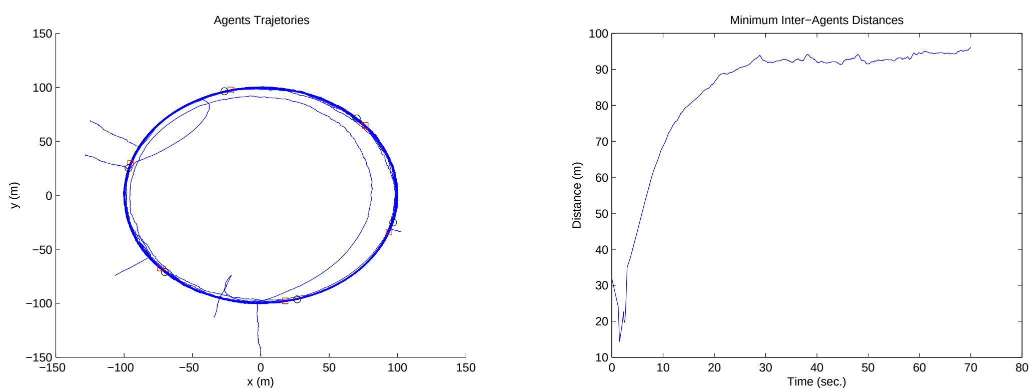
Scenario 4: Not identical but close speeds for each agent. In this scenario we consider the case where $N = 5$, $L_g = 50m$, and the speeds are not identical, i.e. $\bar{v}_1 = 30m/s$, $\bar{v}_2 = 29.8m/s$, $\bar{v}_3 = 30.1m/s$, $\bar{v}_4 = 29.9m/s$, and $\bar{v}_5 = 29.9m/s$. Agent trajectories are depicted in Fig. 8(a), and the minimum of all inter-agent distances at each time is depicted in Fig. 8(b). As is evident from the figures the agent form a shape close to an equilateral polygon and avoid having collision with each other.



(a) The agent trajectories. The red squares represent the final position of the agents after 70 seconds of simulation time

(b) Minimum inter-agent distances.

Figure 6. Scenario 2: $N = 6$, $L_g = 100m$, $\bar{v} = 30m/s$

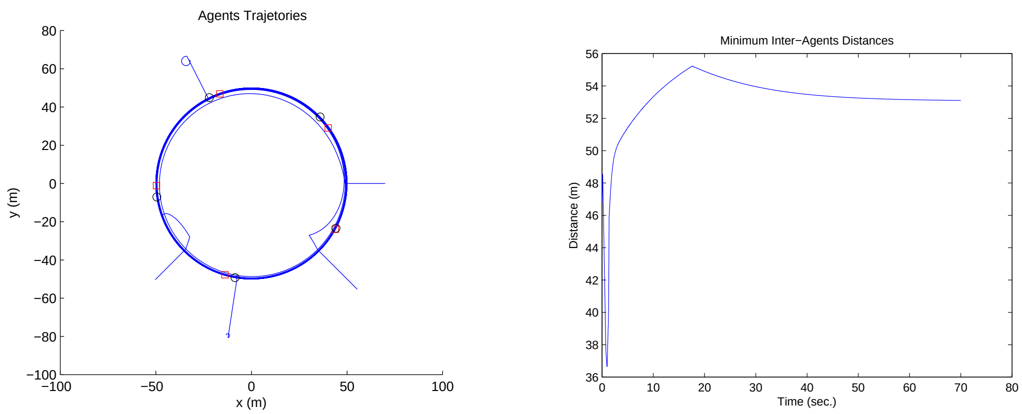


(a)

(b)

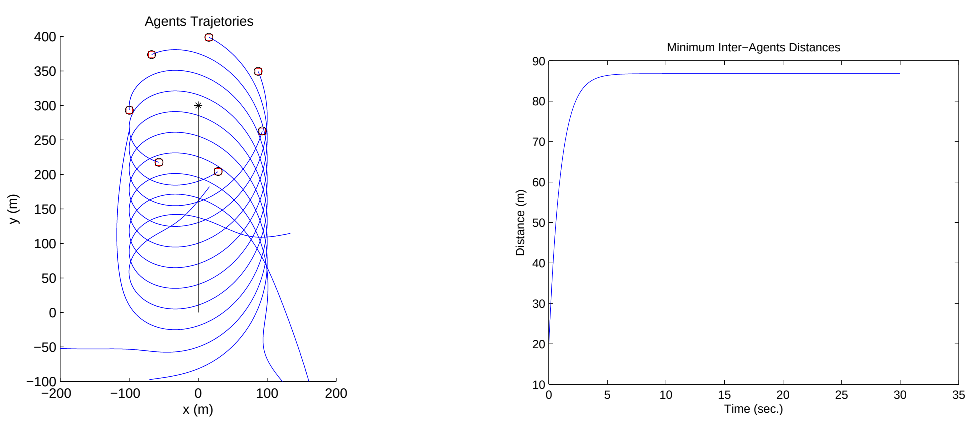
Figure 7. Scenario 3: $N = 6$, $L_g = 100m$, $\bar{v} = 30m/s$, disturbance $\sim N(0, 1)$. (a) The agent trajectories where the motion is perturbed by wind modeled as a Gaussian noise. The red squares represent the final position of the agents after 70 seconds of simulation time. (b) Minimum inter-agent distances where the motion is perturbed by wind modeled as a Gaussian noise.

Scenario 5: Moving Target and Agents with no constant speed constraint In this scenario we consider the case where $N = 7$, $L_g = 100m$, and the speeds are not constant. Agent trajectories are depicted in Fig. 9(a), and the minimum of all inter-agent distances at each time is depicted in Fig. 9(b). As is evident from the figures the agents form an equilateral polygon and avoid having collision with each other.



(a) (b)

Figure 8. Scenario 4: $N = 5$, $L_g = 50m$, $\bar{v}$ different for each agent. (a) The agent trajectories where the agents have different constant speeds. The red squares represent the final position of the agents after 70 seconds of simulation time. (b) Minimum inter-agent distances where the agents have different constant speeds.



(a) (b)

Figure 9. Scenario 5: $N = 7$, $L_g = 100m$, without constant speed constraint. (a) The agent trajectories where the agents o not have constant speeds. The red squares represent the final position of the agents after 30 seconds of simulation time. The black line is the trajectory of the target. (b) Minimum inter-agent distances where the agents do not have constant speeds.

7. Concluding Remarks and Further Research

In this paper the problem of close target reconnaissance by a formation of N agents is considered. The overall close target reconnaissance (CTR) involves subtasks of avoiding inter-agent collisions, reaching a close vicinity of a specified target position, and forming an equilateral polygon formation around the target. The agents performing the task fly at constant speed to model fixed-wing UAVs. A decentralized control scheme is developed for this overall task. The proposed method in this paper is fully analyzed and its finite-time convergence is established. Moreover, note that the algorithm is decentralized in that each agent does not need to know the planned trajectory of any other agent in order to guarantee collision avoidance. However, agents do need to know the location of every other agent. Furthermore, some simulation test results with and without disturbance (wind), are provided to illustrate the performance of the proposed method. Moreover, we have shown a simulation result for the case where the speeds of the agents are constant but not necessarily equal to each other's.

A future research direction is to enforce minimum turning radius constraint on the agents and formally do the analysis under this constraint.

8. Acknowledgments

This work is supported by NICTA, which is funded by the Australian Government as represented by the Department of Broadband, Communications and the Digital Economy and the Australian Research Council through the ICT Centre of Excellence program.

REFERENCES

1. Dixon SR, Wickens CD. Control of multiple UAVs-a workload analysis. *Proceedings 12th International Symposium Aviation Psychology*, Dayton, Ohio, 2003.
2. Finn A, Kabacinski K, Drake SP. Design challenges for an autonomous cooperative of uavs. *Proceedings Information, Decision and Control*, Adelaide, Australia, 2007.
3. Gu G, Chandler P, Schumacher C, Sparks A, Pachter M. Optimal cooperative sensing using a team of UAVs. *IEEE Transactions on Aerospace and Electronic Systems* 2006; **42**(4):1446–1458.
4. Bishop AN, Fidan B, Anderson BDO, Dogancay K, Pathirana PN. Optimality analysis of sensor-target localization geometries. *Automatica* March 2010; **46**(3):479–492.
5. Martinez S, Bullo F. Optimal sensor placement and motion coordination for target tracking. *Journal of Guidance, Control, and Dynamics* 2006; **42**:661–668.
6. Kim TH, Sugie T. Cooperative control for target-capturing task based on a cyclic pursuit strategy. *Automatica* 2007; **43**:1426–1431.
7. Summers TH, Akella MR, Mears MJ. Coordinated standoff tracking of moving targets: Control laws and information architectures. *Journal of Guidance, Control, and Dynamics* 2009; **32**.
8. Sepulchre R, Paley DA, Leonard NE. Stabilization of planar collective motion: All-to-all communication. *IEEE Transactions on Automatic Control* 2006; .
9. Shames I, Fidan B, Anderson BDO. Close target reconnaissance using autonomous UAV formations. *Proceedings of 47th IEEE Conference on Decision and Control*, Cancun, Mexico, 2008; 2326–233.
10. Ledger D. Electronic warfare capabilities of mini uavs. Aerosonde (online: http://www. aerosonde. com/drawarticle/73# finn).
11. Rimon E. A navigation function for a simple rigid body. *Proceedings of IEEE International Conference on Robotics and Automation*, 1991.
12. Rimon E, Koditschek DE. Exact robot navigation using artificial potential functions. *IEEE Transactions on Robotics and Automation* Dec 1992; **8**:501–518.

13. Gennaro MCD, Jadbabaie A. Formation control for a cooperative multi-agent system using decentralized navigation functions. *Proceedings of the 2006 American Control Conference Minneapolis*, Minneapolis, Minnesota, USA, 2006.
14. Tanner HG, Kumar A. Formation stabilization of multipleagents using decentralized navigation functions. *Robotics: Science and Systems*, Boston, 2005; 49–56.
15. Tanner HG, Kumar A. Towards decentralization of multi-robot navigation functions. *Proceedings of the 2005 IEEE International Conference on Robotics and Automation*, Barcelona, Spain, 2005.
16. Lindhe M, Ogren P, Johansson KH. Flocking with obstacle avoidance: A new distributed coordination algorithm based on voronoi partitions. *Proceedings of the 2005 IEEE International Conference on Robotics and Automation*, Barcelona, Spain, 2005; 1785–1790.
17. Ogren P, Leonard NE. A tractable convergent dynamic window approach to obstacle avoidance. *Proceedigns of IEEE/RS International Conference on Intelligent Robots and Systems*, Lausanne, Switzerland, 2002.
18. Soukieh R, Shames I, Fidan B. Obstacle avoidance of robotic formations based on fluid mechanical modeling. *Proc. European Control Conference*, Budapest, Hungary, 2009; 3263–3268.
19. Soukieh R, Shames I, Fidan B. Obstacle avoidance of non-holonomic unicycle robots based on fluid mechanical modeling. *Proc. European Control Conference*, Budapest, Hungary, 2009; 3269–3274.
20. Balch T, Arkin RC. Behavior-based formation control for multirobot teams. *IEEE Transactions on Robotics and Automation* Dec 1998; **13**:926–939.
21. Dougherty R, Ochoa V, Randies Z, Kitts C. A behavioral control approach to formation-keeping through an obstacle field. *Proceedings of IEEE Aerospace Conference*, Big Sky, MT, USA, 2004.
22. Egerstedt M, Xiaoming H. Formation constrained multi-agent control. *IEEE Transactions on Robotics and Automation* Dec 2001; **17**:947 –951.
23. Yu C, Fidan B, Shames I, Sandeep S, Anderson BDO. Collision free coordination of autonomous multiagent system. *European Control Conference, Athens*, Kos, Greece, 2007; 900–907.
24. Mudie I, Melhuish C, Winfield A. Ongoing experiments in autonomous 2d shape formation, with a view to developing autonomous 3d formations with unmanned dirigibles. *Technical Report BS16 1QY*, Intelligent Autonomous Systems Engineering Laboratory, University of the West of England 2001.
25. Balch T, Hybinetre M. Social potentials for scalable multi-robot formations. *Proceedings of 2000 IEEE Conference on Robotics and Automation*, San Francisco, CA, USA, 2000; 73–80.
26. Shames I, Yu C, Fidan B, Anderson B. Externally excited coordination of autonomous formations. *Mediterranean Conference on Control & Automation*, 2007; T33–003.
27. Hoffmann G, Tomlin C. Decentralized cooperative collision avoidance for acceleration constrained vehicles. *47th IEEE Conference on Decision and Control, 2008. CDC 2008*, 2008; 4357–4363.
28. Mastellone S, Stipanovic D, Graunke C, Intlekofer K, Spong M. Formation control and collision avoidance for multi-agent non-holonomic systems: Theory and experiments. *The International Journal of Robotics Research* 2008; **27**(1):107.
29. Dimarogonas D, Loizou S, Kyriakopoulos K, Zavlanos M. A feedback stabilization and collision avoidance scheme for multiple independent non-point agents. *Automatica* 2006; **42**(2):229–243.
30. Kuhn HW. The hungarian method for the assignment problem. *50 Years of Integer Programming 1958-2008* 2008; :29–47.
31. Krasovskii NN. *Stability of Motion, Application of Lyapunov's Second Method to Differential Systems and Equations With Delay.* Stanford University Press: Stanford California, 1963.

APPENDIX

I. Proof of Proposition 1

Proof of 1. For for $2(\bar{k}-1)\pi L_g \leq |\|p_i(t)\| - L_g| \leq 2\bar{k}\pi L_g$, $\bar{k} \in \mathbb{Z}$ and $\bar{k} > 0$ we have

$$F_{i,j}(t) = \big|\|p_i(t)\| - L_g - 2(\bar{k}-1)\pi L_g - L_g\alpha_{ji}(t)\big| \tag{23}$$

We prove the first statement, via obtaining a contradiction. To this end suppose there is an agent pair $(\tilde{A}_j, \tilde{A}_k)$ that satisfies the aforementioned inequalities:

$$\begin{aligned} F_{i,j}(t) &= |\Delta L_i(t) - L_g\alpha_{ji}(t)| = \varepsilon_j < \varepsilon_\alpha \\ F_{i,k}(t) &= |\Delta L_i(t) - L_g\alpha_{ki}(t)| = \varepsilon_k < \varepsilon_\alpha . \end{aligned} \tag{24}$$

Without loss of generality assume $\varepsilon_j > \varepsilon_k$. Considering first the case where $\Delta L_i(t) - L_g\alpha_{ji}(t)$ and $\Delta L_i(t) - L_g\alpha_{ki}(t)$ have the same sign, (24) implies that

$$\begin{aligned} F_{i,j}(t) - F_{i,k}(t) &= |\Delta L_i(t) - L_g\alpha_{ji}(t)| - |\Delta L_i(t) - L_g\alpha_{ki}(t)| = \varepsilon_j - \varepsilon_k \\ &= L_g| - \alpha_{ji}(t) + \alpha_{ki}(t)| < \varepsilon_\alpha \end{aligned} \tag{25}$$

On the other hand, by definition of $\alpha_{ji},\ \alpha_{ki},\ \tilde{\theta}_{kj}$ we have $\widetilde{\theta}_{kj} \le | - \alpha_{ji}(t) + \alpha_{ki}(t)|$, which together with (25) implies

$$L_g\widetilde{\theta}_{kj} < \varepsilon_\alpha \tag{26}$$

which contradicts Assumption 4.

Next, without loss of generality, assume $F_{i,k}(t) > 0$ and consider the case where $\Delta L_i(t) - L_g\alpha_{ji}(t)$ and $\Delta L_i(t) - L_g\alpha_{ki}(t)$ have different signs. we have

$$\begin{aligned} F_{i,j}(t) + F_{i,k}(t) &= |\Delta L_i(t) - L_g\alpha_{ji}(t)| + |\Delta L_i(t) - L_g\alpha_{ki}(t)| = \varepsilon_j + \varepsilon_k \\ &= L_g(\alpha_{ji}(t) - \alpha_{ki}(t)) < 2\varepsilon_\alpha. \end{aligned} \tag{27}$$

Thus

$$F_{i,j}(t) + F_{i,k}(t) = L_g(\alpha_{ji}(t) - \alpha_{ki}(t)) < 2\varepsilon_\alpha. \tag{28}$$

Again using the inequality $\widetilde{\theta}_{kj} \le |\alpha_{ji}(t) - \alpha_{ki}(t)|$ we have

$$L_g\widetilde{\theta}_{kj} < 2\varepsilon_\alpha \tag{29}$$

which contradicts with Assumption 4. Hence, there is no pair agent $(\tilde{A}_j, \tilde{A}_k)$ that both satisfies $|\Delta L_i(t) - L_g\alpha_{ji}(t)| < \varepsilon_\alpha$ and $|\Delta L_i(t) - L_g\alpha_{ki}(t)| < \varepsilon_\alpha$.

Proof of 2. First let us show that $F_{i,j}(t) = F_{i,j}(\sigma_i\bar{\tau}_i)$ for any $t \ge \sigma_i\bar{\tau}_i$. Noting that the speeds of agents A_i and $\tilde{A}_j$ are both $\bar{v}$, for any $t = \sigma_i\bar{\tau}_i + \Delta t \ge \sigma_i\bar{\tau}_i$, by (10), we have

$$\begin{aligned} \Delta L_i(t) &= (\|p_i(t)\| - L_g) \mod 2\pi L_g \\ &= (\|p_i(\sigma_i\bar{\tau}_i)\| - \bar{v}\Delta t - L_g) \mod 2\pi L_g \\ &= (\Delta L_i(\sigma_i\bar{\tau}_i) - \bar{v}\Delta t) \mod 2\pi L_g \end{aligned}$$

and

$$L_g\alpha_{j^*i}(t) = (L_g\alpha_{j^*i}(\sigma_i\bar{\tau}_i) - L_g\frac{\bar{v}}{L_g}t) \mod 2\pi L_g.$$

Therefore,

$$\Delta L_i(t) - L_g\alpha_{j^*i}(t) = (\Delta L_i(\sigma_i\bar{\tau}_i) - L_g\alpha_{j^*i}(\sigma_i\bar{\tau}_i)) \mod 2\pi L_g.$$

Hence $F_{i,j}(t) = F_{i,j}(\sigma_i\bar{\tau}_i)$.

Now to establish the inequality we need to show that $F_{i,j}(\sigma_i\bar{\tau}_i) \ge 0$. For the case $\sigma_i = 0$, the result is straightforward. For $\sigma_i = 1$, because of (6)-(8) we have $p_i(\bar{\tau}_i) = p_i(0)$ and hence

$$\Delta L_i(\bar{\tau}_i) = \Delta L_i(0).$$

Furthermore by definition of $\bar{\tau}_i$, we have

$$L_g\alpha_{j^*i}(\bar{\tau}_i) = L_g\alpha_{j^*i}(0) - \Delta_i$$

Therefore

$$\Delta L_i(\bar{\tau}_i) - L_g\alpha_{j^*i}(\bar{\tau}_i) = \Delta L_i(0) - L_g\alpha_{j^*i}(0) + \Delta_i = \varepsilon_\alpha$$

because of (4). Therefore $F_{i,j}(\sigma_i\bar{\tau}_i) = \varepsilon_\alpha$.